

## Unit 3 – Linear and Exponential Functions

### Example:

Every year Silas buys fudge at the state fair. He buys peanut butter and chocolate. This year he intends to buy \$24 worth of fudge. If chocolate costs \$4 per pound and peanut butter costs \$3 per pound, what are the different combinations of fudge that he can purchase if he only buys whole pounds of fudge?

### Example:

Silas decides he does not have to spend exactly \$24 on the fudge, but he will not spend more than \$24. What are the different combinations of fudge purchases he can make?

- 1) Consider the equations  $y = 2x - 3$  and  $y = -x + 6$ .
  - a. Complete the tables below, and then graph the equations on the same coordinate axes.

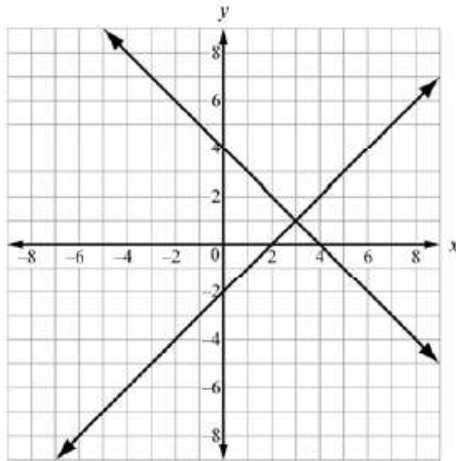
$y = 2x - 3$	
$x$	$y$

$y = -x + 6$	
$x$	$y$

- b. Is there an ordered pair that satisfies both equations? If so, what is it?
    - c. Graph both equations on the same coordinate plane by plotting the ordered pairs from the tables and connecting the points.
    - d. Do the lines appear to intersect? If so, where? How can you tell that the point where the lines appear to intersect is a common point for both lines?
- 2) Graph the inequality  $x + 2y < 4$ .

EOCT Practice Items

1) Two lines are graphed on this coordinate plane.



Which point appears to be a solution of the equations of both lines?

- A. (0, -2)
- B. (0, 4)
- C. (2, 0)
- D. (3, 1)

2) Based on the tables, at what point do the lines  $y = -x + 5$  and  $y = 2x - 1$  intersect?

$y = -x + 5$	
$x$	$y$
-1	6
0	5
1	4
2	3
3	2

$y = 2x - 1$	
$x$	$y$
-1	-3
0	-1
1	1
2	3
3	5

- A. (1, 1)
- B. (3, 5)
- C. (2, 3)
- D. (3, 2)

**Example:**

Given  $f(x) = 2x - 1$ , find  $f(7)$ .

**Example:**

If  $g(6) = 3 - 5(6)$ , what is  $g(x)$ ?

**Example:**

If  $f(-2) = -4(-2)$ , what is  $f(b)$ ?

**Example:**

Graph  $f(x) = 2x - 1$ .

**Example:**

Consider the sequence: 3, 6, 9, 12, 15, . . . The first term is 3, the second term is 6, the third term is 9, and so on. The “. . .” at the end of the sequence indicates the pattern continues without end. Can this pattern be considered a function?

**REVIEW EXAMPLES**

- A manufacturer keeps track of her monthly costs by using a “cost function” that assigns a total cost for a given number of manufactured items,  $x$ . The function is  $C(x) = 5,000 + 1.3x$ .
  - What is the domain of the function?
  - What is the cost of 2,000 items?
  - If costs must be kept below \$10,000 this month, what is the greatest number of items she can manufacture?
- Consider the first six terms of this sequence: 5, 7, 11, 19, 35, 67, . . .
  - What is  $a_1$ ? What is  $a_3$ ?
  - If the sequence defines a function, what is the range?
- The function  $f(n) = -(1 - 4n)$  represents a sequence. Create a table showing the first five terms in the sequence. Identify the domain and range of the function.

**EOCT Practice Items**

- 1) The first term in this sequence is  $-1$ .

$n$	1	2	3	4	5	. . .
$a_n$	$-1$	1	3	5	7	. . .

Which function represents the sequence?

- $a_n = a_{n-1} + 1$
- $a_n = a_{n-1} + 2$
- $a_n = 2a_{n-1} - 1$
- $a_n = 2a_{n-1} - 3$

- 2) Which function is modeled in this table?

$x$	$f(x)$
1	8
2	11
3	14
4	17

- $f(x) = x + 7$
- $f(x) = x + 9$
- $f(x) = 2x + 5$
- $f(x) = 3x + 5$

3) Which explicit formula describes the pattern in this table?

$d$	$C$
2	6.28
3	9.42
5	15.70
10	31.40

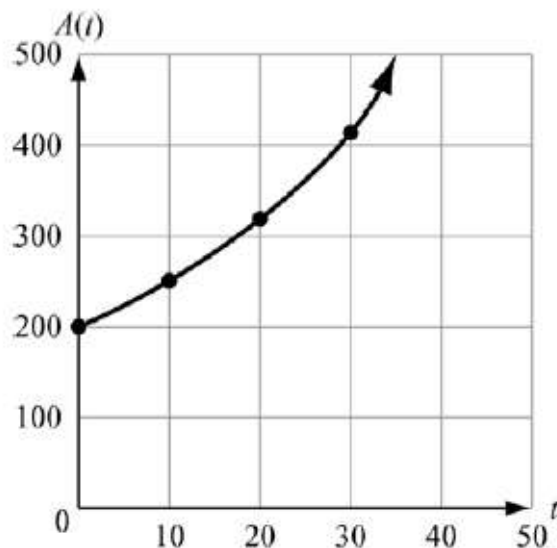
- A.  $d = 3.14 \times C$
- B.  $3.14 \times C = d$
- C.  $31.4 \times 10 = C$
- D.  $C = 3.14 \times d$

4) If  $f(12) = 4(12) - 20$ , which function gives  $f(x)$ ?

- A.  $f(x) = 4x$
- B.  $f(x) = 12x$
- C.  $f(x) = 4x - 20$
- D.  $f(x) = 12x - 20$

#### REVIEW EXAMPLES

- 1) The amount accumulated in a bank account over a time period  $t$  and based on an initial deposit of \$200 is found using the formula  $A(t) = 200(1.025)^t$ ,  $t \geq 0$ . Time,  $t$ , is represented on the horizontal axis. The accumulated amount,  $A(t)$ , is represented on the vertical axis.



- a. What are the intercepts of the function?
  - b. What is the domain of the function?
  - c. Why are all the  $t$  values non-negative?
  - d. What is the range of the function?
  - e. Does the function have a maximum or minimum value?
- 2) A company uses the function  $V(x) = 28,000 - 1,750x$  to represent the depreciation of a truck, where  $V(x)$  is the value of the truck and  $x$  is the number of years after its purchase. Use the table of values shown below.

$x$ , years	$V(x)$ , value in \$
0	28,000
1	26,250
2	24,500
3	22,750
4	21,000
5	19,250

- a. What is the  $y$ -intercept of the graph of the function?
- b. Does the graph of the function have an  $x$ -intercept?
- c. Does the function increase or decrease?

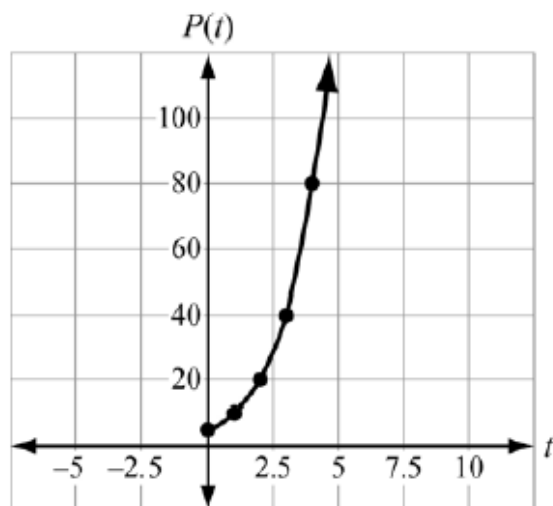
### **EOCT Practice Items**

- 1) **A farmer owns a horse that can continuously run an average of 8 miles an hour for up to 6 hours. Let  $y$  be the distance the horse can travel for a given  $x$  amount of time in hours. The horse's progress can be modeled by a function.**

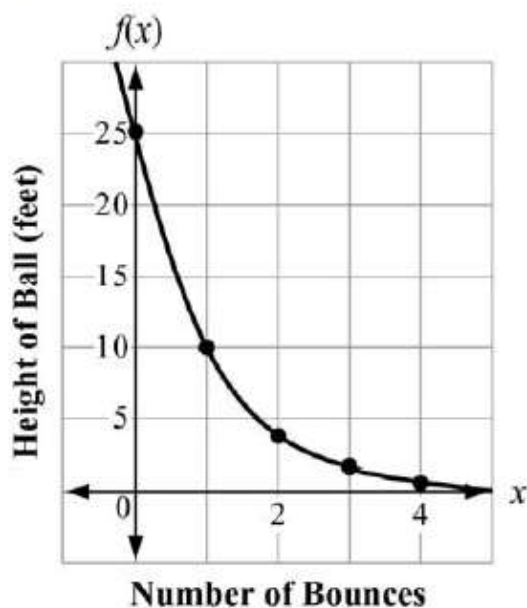
**Which of the following describes the domain of the function?**

- A.  $0 \leq x \leq 6$
- B.  $0 \leq y \leq 6$
- C.  $0 \leq x \leq 48$
- D.  $0 \leq y \leq 48$

- 2) A population of squirrels doubles every year. Initially there were 5 squirrels. A biologist studying the squirrels created a function to model their population growth,  $P(t) = 5(2^t)$  where  $t$  is time. The graph of the function is shown. What is the range of the function?



- A. any real number  
 B. any whole number greater than 0  
 C. any whole number greater than 5  
 D. any whole number greater than or equal to 5
- 3) The function graphed on this coordinate grid shows  $f(x)$ , the height of a dropped ball in feet after its  $x$ th bounce.



- A. bounce 1  
 B. bounce 2  
 C. bounce 3  
 D. bounce 4

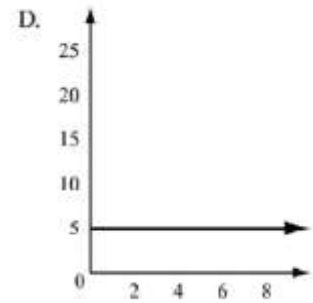
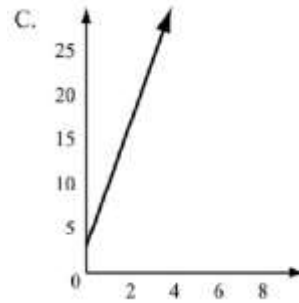
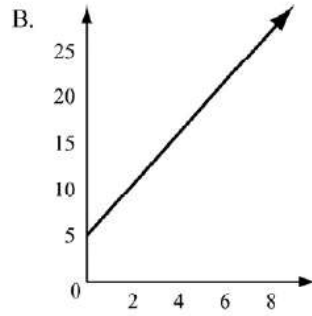
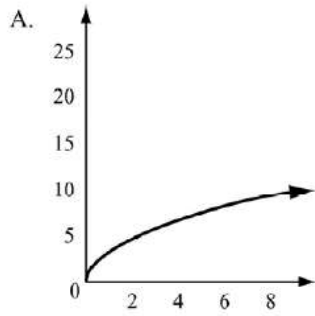
On which bounce was the height of the ball 10 feet?

## REVIEW EXAMPLES

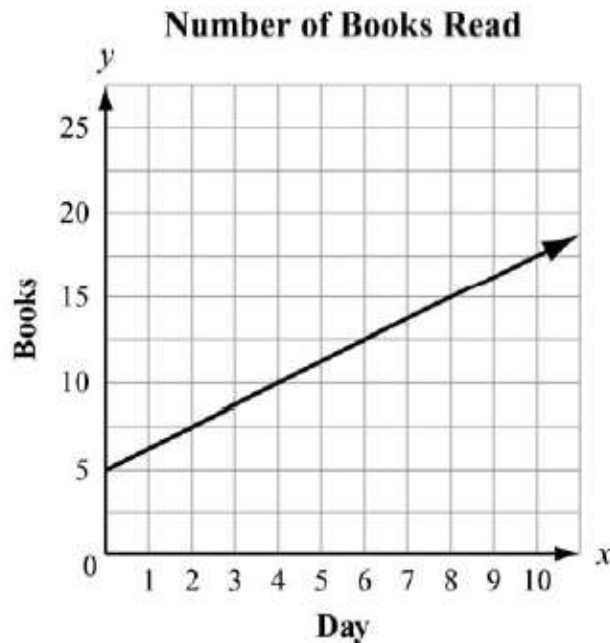
- 1) What are the key features of the function  $p(x) = \frac{1}{2}x - 3$ ?

### EOCT Practice Items

- 1) To rent a canoe, the cost is \$3 for the oars and life preserver, plus \$5 an hour for the canoe. Which graph models the cost of renting a canoe?



- 2) Juan and Patti decided to see who could read the most books in a month. They began to keep track after Patti had already read 5 books that month. This graph shows the number of books Patti read for the next 10 days.





If Juan has read no books before the fourth day of the month and he reads at the same rate as Patti, how many books will he have read by day 12?

- A. 5
- B. 10
- C. 15
- D. 20

### REVIEW EXAMPLES

- 1) The terms of a sequence increase by a constant amount. If the first term is 7 and the fourth term is 16:
- a. List the first six terms of the sequence.
  - b. What is the explicit formula for the sequence?
  - c. What is the recursive rule for the sequence?

### EOCT Practice Items

- 1) Which function represents this sequence?

$n$	1	2	3	4	5	...
$a_n$	6	18	54	162	486	...

- A.  $f(n) = 3^{n-1}$
  - B.  $f(n) = 6^{n-1}$
  - C.  $f(n) = 3(6^{n-1})$
  - D.  $f(n) = 6(3^{n-1})$
- 2) The first term in this sequence is 3.

$n$	1	2	3	4	5	...
$a_n$	3	10	17	24	31	...

Which function represents the sequence?

- A.  $f(n) = n + 3$
- B.  $f(n) = 7n - 4$
- C.  $f(n) = 3n + 7$
- D.  $f(n) = n + 7$



3) The points  $(0, 1)$ ,  $(1, 5)$ ,  $(2, 25)$ ,  $(3, 125)$  are on the graph of a function. Which equation represents that function?

A.  $f(x) = 2^x$

B.  $f(x) = 3^x$

C.  $f(x) = 4^x$

D.  $f(x) = 5^x$

### REVIEW EXAMPLES

1) For the function  $f(x) = 3^x$ :

a. Find the function that represents a 5 unit translation upward of the function.

b. Is the function even, odd, or neither even nor odd?

2) Given the function  $f(x) = 3x + 4$ :

a. Compare  $f(x)$  to  $3f(x)$ .

b. Compare  $f(x)$  to  $f(3x)$ .

c. Draw the graph of  $-f(x)$ .

d. Which has the fastest growth rate:  $f(x)$ ,  $3f(x)$ , or  $-f(x)$ ?

### EOCT Practice Items

1) A function  $g$  is an odd function. If  $g(-3) = 4$ , which other point lies on the graph of  $g$ ?

A.  $(3, -4)$

B.  $(-3, -4)$

C.  $(4, -3)$

D.  $(-4, 3)$

2) Which statement is true about the function  $f(x) = 7^x$ ?

A. The function is odd because  $-f(x) = f(-x)$ .

B. The function is even because  $-f(x) = f(-x)$ .

C. The function is odd because  $f(x) = f(-x)$ .

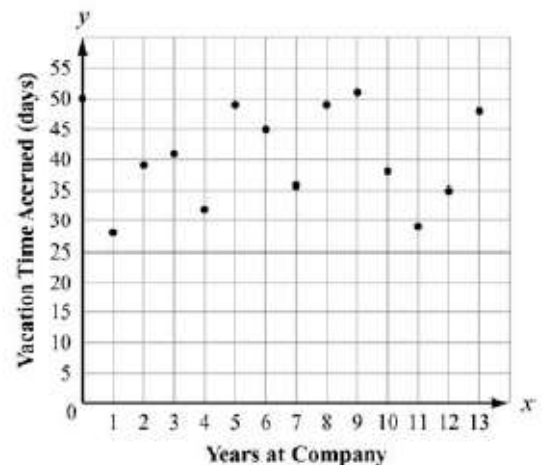
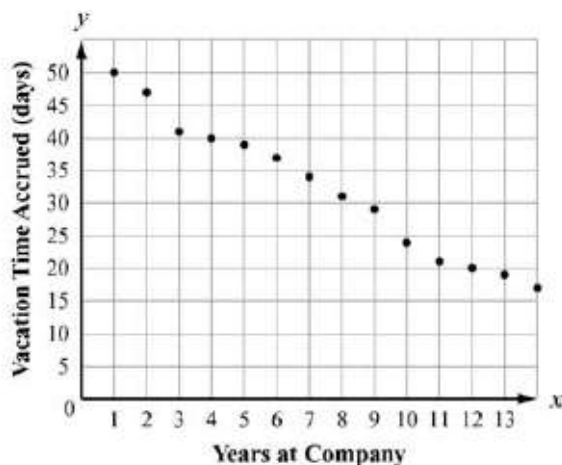
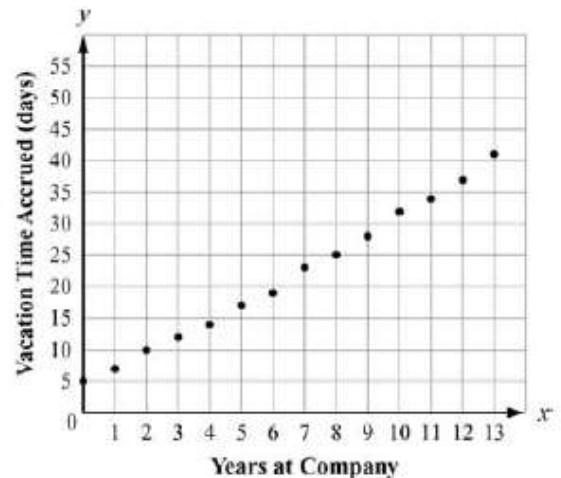
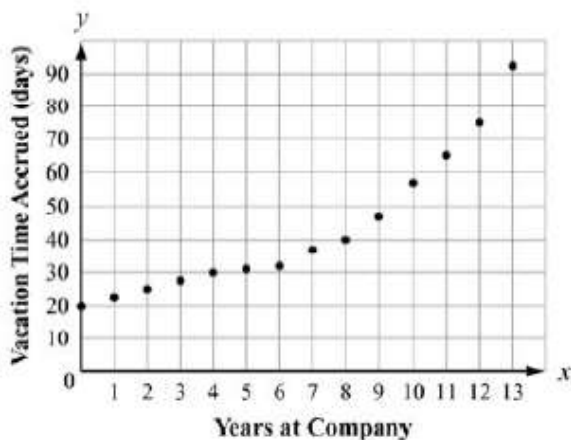
D. The function is even because  $f(x) = f(-x)$ .

## REVIEW EXAMPLES

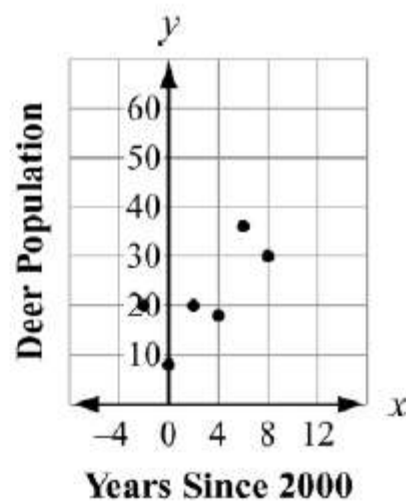
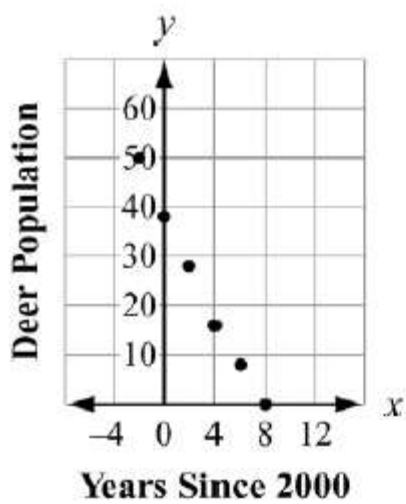
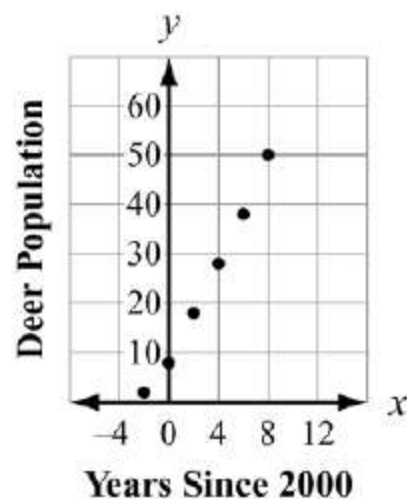
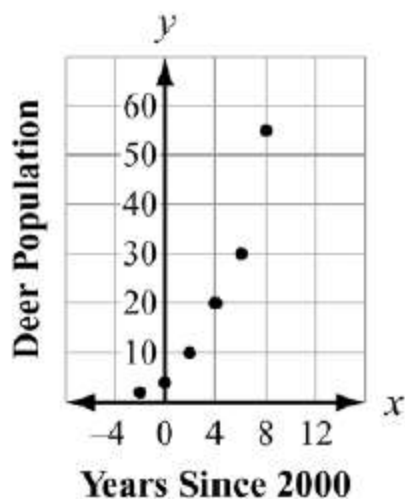
- The swans on Elsworth Pond have been increasing in number each year. Felix has been keeping track and so far he has counted 2, 4, 7, 17, and 33 swans each year for the past five years.
  - Make a scatter plot of the swan population.
  - What type of model would be a better fit, linear or exponential? Explain your answer.
  - How many swans should Felix expect next year if the trend continues? Explain your answer.
- Given the sequence 7, 10, 13, 16, . . .
  - Does it appear to be linear or exponential?
  - Determine a function to describe the sequence.
  - What would the 20th term of the sequence be?

## EOCT Practice Items

- Which scatter plot BEST represents a model of linear growth?



2) Which scatter plot BEST represents a model of exponential growth?



3) Which table represents an exponential function?

A.

$x$	0	1	2	3	4
$y$	5	6	7	8	9

B.

$x$	0	1	2	3	4
$y$	0	22	44	66	88

C.

$x$	0	1	2	3	4
$y$	5	13	21	29	37

D.

$x$	0	1	2	3	4
$y$	0	3	9	27	81

## REVIEW EXAMPLES

- 1) Alice finds her flower bulbs multiply each year. She started with just 24 tulip plants. After one year she had 72 plants. Two years later she had 120. Find a linear function to model the growth of Alice's bulbs.
- 2) Suppose Alice discovers she counted wrong the second year and she actually had 216 tulip plants. She realizes the growth is not linear because the rate of change was not the same. She must use an exponential model for the growth of her tulip bulbs. Find the exponential function to model the growth.

### EOCT Practice Item

- 1) If the parent function is  $f(x) = mx + b$ , what is the value of the parameter  $m$  for the line passing through the points  $(-2, 7)$  and  $(4, 3)$ ?
  - A.  $-9$
  - B.  $-\frac{3}{2}$
  - C.  $-2$
  - D.  $-\frac{2}{3}$